Improved Deep Embeddings for Inferencing with Multi-Layered Graphs

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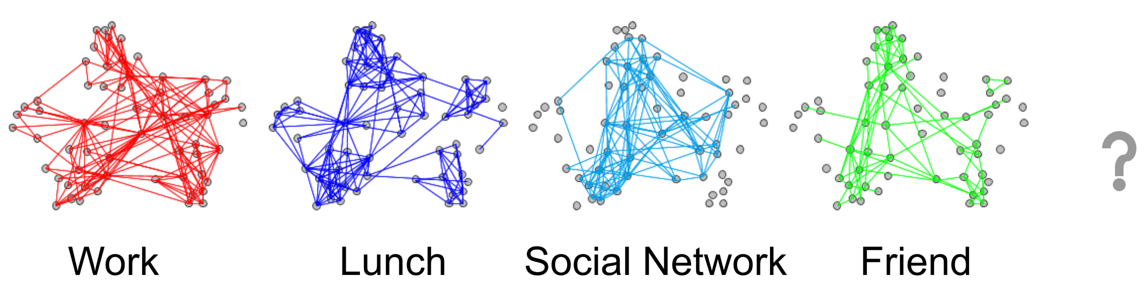
Introduction

- **Problem:** Inferencing with multi-layered graphs – different relational structure exists in each layer for the same set of nodes.
- **Goal:** Obtaining concise embeddings that preserve the multi-view relationships.

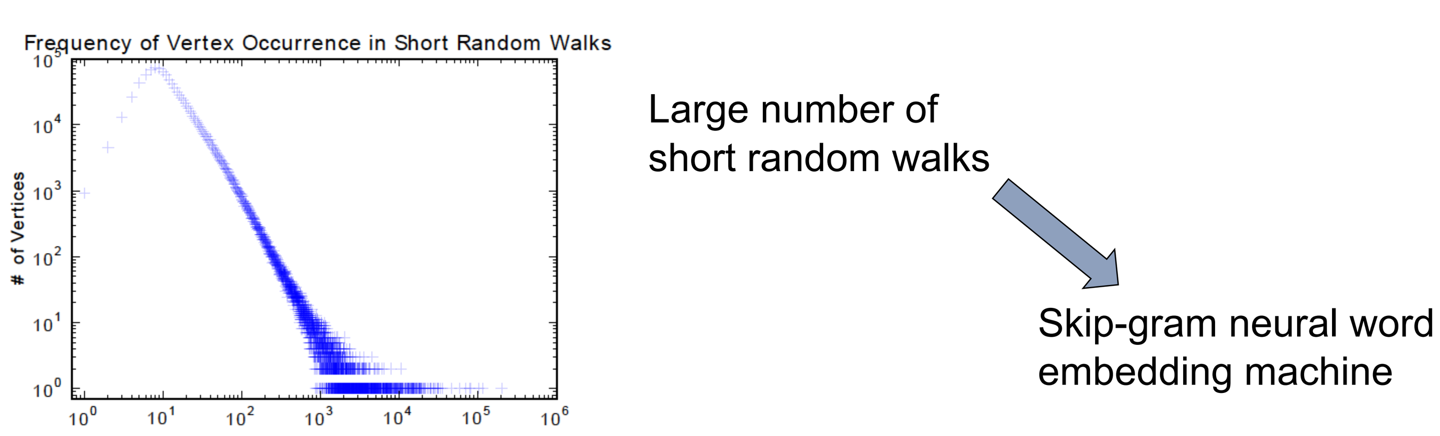


Node embeddings for inferencing tasks:
- node classification
- link prediction
- community detection

- **Challenges:** Heterogeneity in the relationship types, varying levels of sparsity in different layers and scalability with increasing number of layers.
- **Solution:** A scalable embedding approach based on Deepwalk-style optimization and refinement to encourage cohesive community formation.

Prior Art

- Single-layer graph embedding algorithms:
  - Scalable approaches based on distributional hypothesis [1]



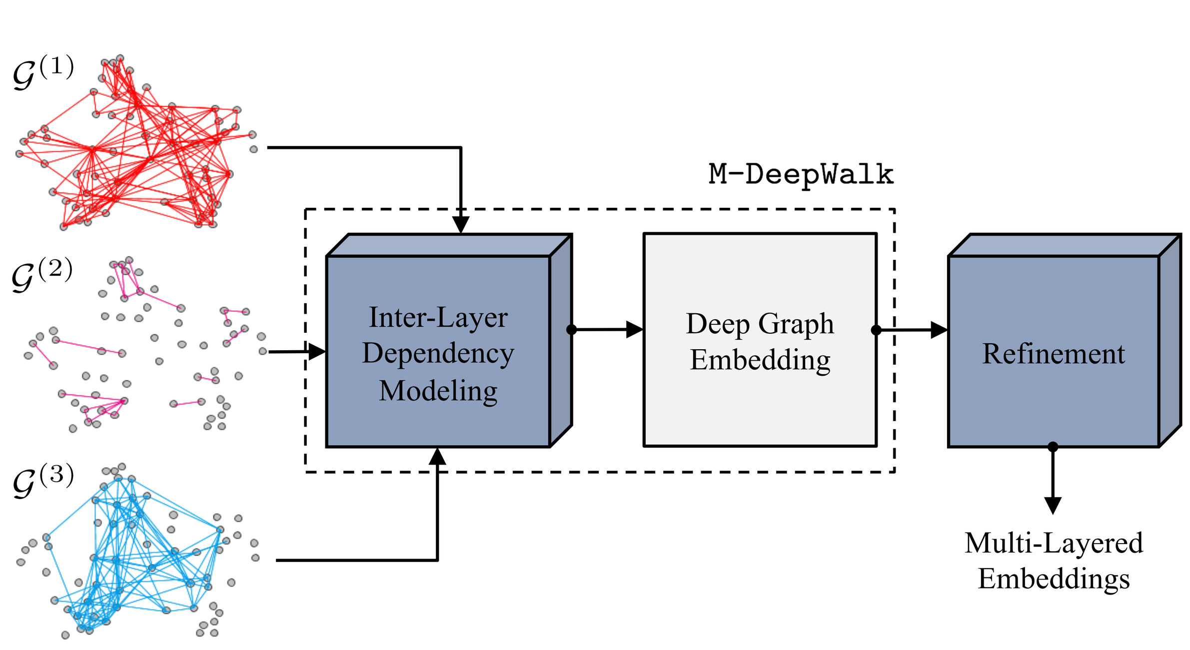
- Multi-layered graph embedding algorithms:
  - Modularity-based approach for community detection [2]

$$Q_{\text{multi}} = \frac{1}{2\mu}\sum_{i,j}\sum_{d,r}[\underbrace{(A_{ij}^{d} - \gamma_d \frac{k_i^d k_j^d}{2m_d})\delta(d,r)}_{\text{Intra-layer contribution}} + \underbrace{\delta(i,j)\sigma_j^{d,r}}_{\text{Inter-layer contribution}}]\delta(g_i^d, g_j^r)$$

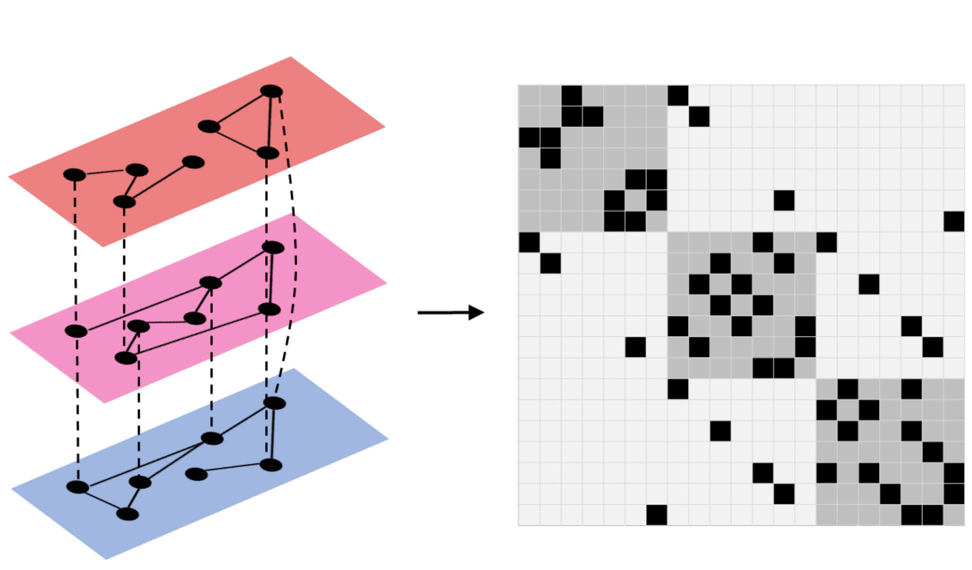
  - Scalable multiplex network embedding [3]

Common embedding for corresponding nodes + Additional embedding to capture aspect of each layer

Stage 1: M-DeepWalk



- Explicitly model inter-layer dependency by constructing supra-graph and then perform M-DeepWalk to learn the embeddings



Define inter-layer edges based on Jaccard Coefficient:

$$e_{ij}^{(l,m)} = 0, \text{if } i \neq j,$$

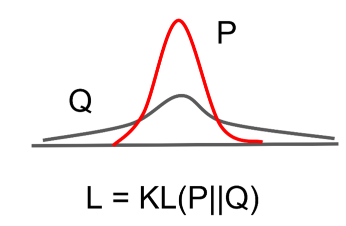
$$e_{ij}^{(l,m)} = \frac{|\mathcal{N}_i^{(l)} \cap \mathcal{N}_j^{(m)}|}{|\mathcal{N}_i^{(l)} \cup \mathcal{N}_j^{(m)}|}, \text{if } i = j$$

Stage 2: Refinement

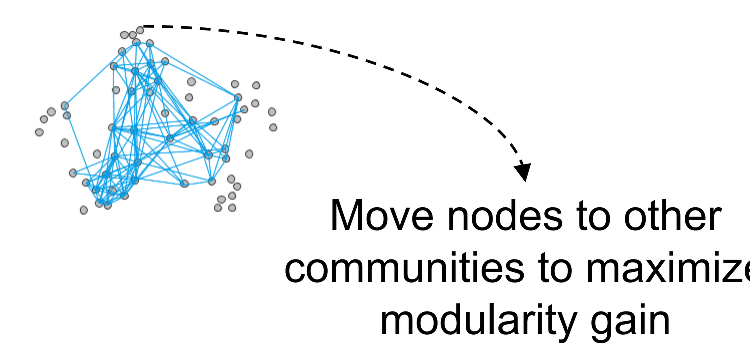
- A refinement stage to fine-tune the learned embeddings based on two loss terms:

Proxy clustering loss

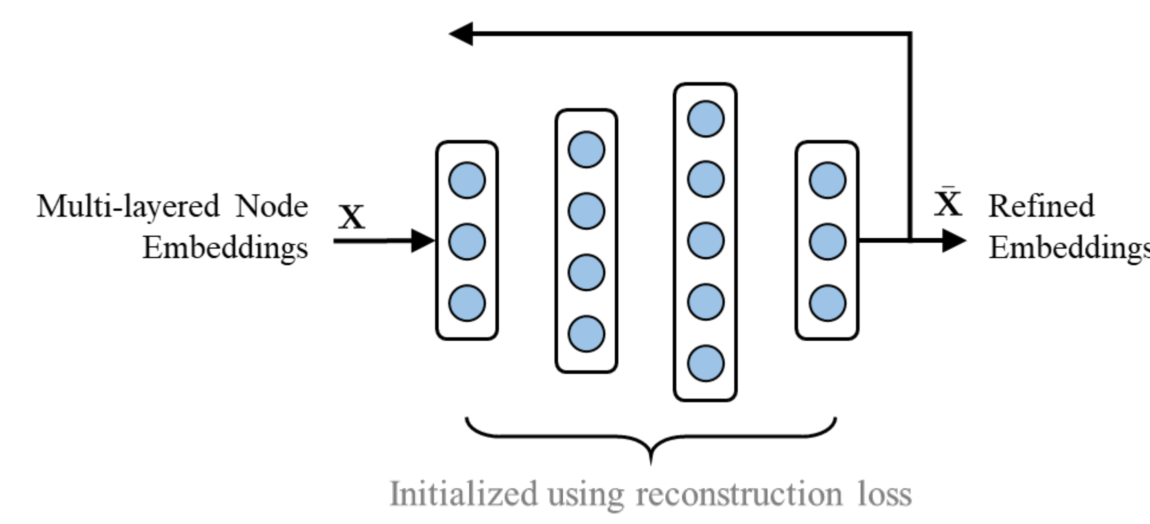
$$p_{ij} = \frac{q_{ij}^2 / f_j}{\sum_{j'} q_{ij'}^2 / f_j}$$



Multi-Slice Modularity Loss



- Obtain refined embeddings using the proposed losses – Embeddings are initialized using a simple reconstruction loss



Experimental Results

Task 1: Node Classification

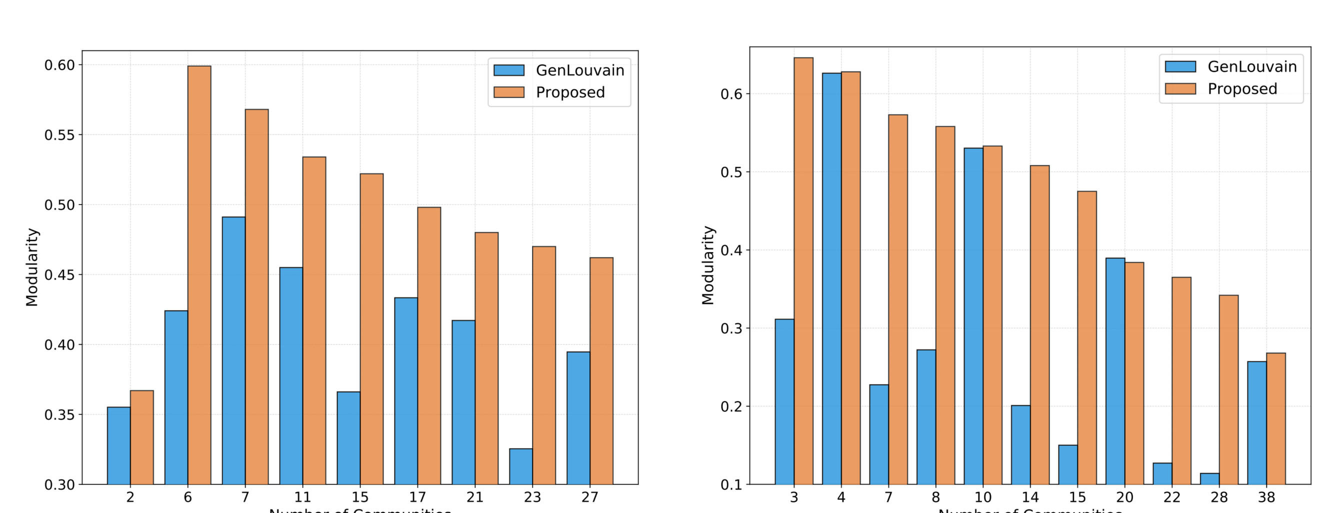
Dataset / Accuracy (%)	Method					
	DeepWalk	Node2Vec	PMNE	MNE	Proposed (w/o refine)	Proposed (w/ refine)
Leskovec-Ng	99.2	96.3	94.5	92.4	99.7	**100**
Reinnovation	74.7	76.0	77.4	75.0	76.0	**85.1**
Congress Votes	99.8	92.4	98.4	-	**100**	**100**
Mammography	80.6	80.2	78.5	74.3	81.3	**81.5**
Balance Scale	90.9	89.3	91.1	82.4	81.5	**92.1**

Task 2: Link Prediction

Dataset / AUROC	Method				
	DeepWalk	LINE	Node2Vec	PMNE	Proposed
Leskovec-Ng	**0.84**	0.62	0.71	0.49	**0.84**
Reinnovation	**0.99**	0.78	**0.99**	0.78	**0.99**
Congress Votes	**1.0**	0.99	**1.0**	0.79	**1.0**
Mammography	**1.0**	**1.0**	**1.0**	0.77	**1.0**
Balance Scale	**1.0**	**1.0**	**1.0**	0.82	**1.0**
LAZEGA	0.88	0.69	0.8	0.82	**0.91**
C.ELEGANS	0.93	0.77	0.89	0.75	**0.94**

Task 3: Multi-layered Community Detection

LAZEGA



C.ELEGANS

References

[1] Bryan Perozzi, Rami Al-Rfou, and Steven Skiena, "Deepwalk: Online learning of social representations," KDD 2014.
[2] Peter J Mucha, Thomas Richardson, Kevin Macon, Mason A Porter, and Jukka-Pekka Onnela, "Community structure in time-dependent, multiscale, and multiplex networks," science, vol. 328, no. 5980, pp. 876–878, 2010.
[3] Hongming Zhang, Liwei Qiu, Lingling Yi, and Yangqiu Song, "Scalable multiplex network embedding.," IJCAI 2018.